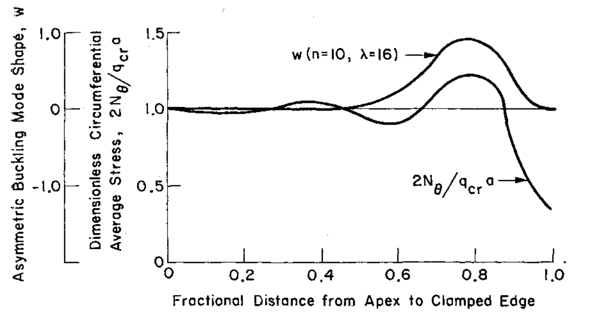

Fig. 2 Boundary-layer effect in circumferential average stress and asymmetric buckling mode.

caps rather than merely showing their existence at bifurcation pressures.

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Equivalence of Nonequilibrium Flows behind Normal and Oblique Shock Waves

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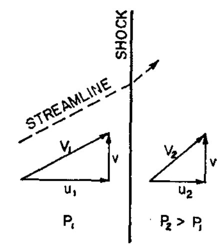
Dissociating flow behind a plane oblique shock wave was investigated by Epstein[1] and has recently been elaborated by Hsu and Anderson[2] using a different approach proposed by Sedney.[3] In both cases numerical integrations are required for each shock-wave angle and each freestream Mach number; the numerical computations in Ref. 2 are comparatively involved. Thus far, an important, fundamental aspect of nonequilibrium flow seems to have been overlooked, namely, that just as in the classical case of equilibrium flow, nonequilibrium flow behind an oblique shock wave with curved streamlines and apparently very nonuniform flow properties is *equivalent* to a nonequilibrium flow behind a normal shock wave. This note serves to point out this equivalence.

To see the equivalence physically, one notes that all flow properties are constant along lines parallel to an oblique shock wave[1] and that the velocity component v parallel to the oblique shock wave remains constant throughout the flow field because of the absence of pressure gradient in the direction parallel to the shock wave. Hence, to an observer moving with the speed v along the shock front (Fig. 1), the entire flow field is identical to the flow field in the case of a normal shock wave. To verify this equivalence mathematically, one merely writes the fundamental differential equations in rectangular coordinates with the y axis parallel to the oblique shock wave, keeping in mind that all flow variables are independent of y. The resulting equations are identical to those for normal shock waves. Incidentally, Epstein would have reached the same conclusion had he adopted the shock-oriented coordinates.

Fig. 1 Equivalence of oblique shock to normal shock to observer moving with velocity v.



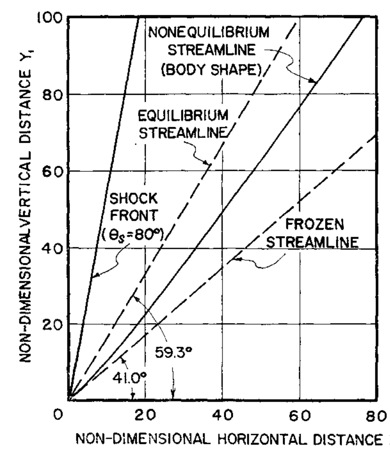


Fig. 2 Streamlines behind an oblique shock wave (θ_s = 80°).

Recognizing this equivalence, one can easily construct solutions for oblique shock waves, given a normal-shock solution. Such plane oblique shock waves with nonequilibrium flow behind them may be considered as being supported by a properly shaped cusped body,[2] or they may be considered as asymptotes to shock waves generated by some two-dimensional bodies, e.g., the wedge.[3] The streamlines (or the shape of the cusped body) can be found through a simple integration; they are necessarily curved since the tangential velocity component remains constant while the normal velocity component varies with the distance normal to the shock wave. A typical streamline constructed by the isocline method is presented in Fig. 2, based on a normal shock solution by Freeman[4] (curve C of Fig. 2a in Ref. 4).

It ought to be noted that this equivalence applies to all nonequilibrium flows, be they dissociating, vibrationally relaxing, or chemically reacting.

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